higher education
& training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE (VOCATIONAL)

NOVEMBER 2010

MATHEMATICS
(First Paper)
NQF LEVEL 4

1 NOVEMBER 2010

This marking guideline consists of 7 pages.

Copyright reserved

This document was downloaded from www.learnsupport.co.za

ACADEMIC SUPPORT
TECHNOLOGIES (PTY) LTD

QUESTION 1

1.1 1.1.1

$2x+1$ is a factor of $f(x)=2x^3-5x^2+x+a$

$\therefore f(-\tfrac{1}{2})=0$ ✓

$2(-\tfrac{1}{2})^3-5(-\tfrac{1}{2})^2+(-\tfrac{1}{2})+a=0$ ✓ Correct substitution

$a=2$ ✓ (3)

1.1.2

$f(x)=2x^3-5x^2+x+2$

$f(1)=2(1)^3-5(1)^2+(1)+2=0 \Rightarrow (x-1)$ is a factor of $f(x)$ ✓

$f(2)=2(2)^3-5(2)^2+(2)+2=0 \Rightarrow (x-2)$ is a factor of $f(x)$ ✓

$f(x)=2x^3-5x^2+x+2=(2x+1)(x-1)(x-2)$ ✓ (3)

Alternate Solution

$f(x)=2x^3-5x^2+x+2=(2x+1)(px^2+qx+r)$

$f(x)=2x^3-5x^2+x+2=(2x+1)(1x^2+qx+2)$ ✓ By inspection

$f(x)=2x^3-5x^2+x+2=(2x+1)(x^2-3x+2)$ ✓ By inspection

$f(x)=2x^3-5x^2+x+2=(2x+1)(x-1)(x-2)$ ✓

Alternate Solution

$f(x)=2x^3-5x^2+x+2=(2x+1)(x^2-3x+2)$ ✓✓ By long division

$f(x)=2x^3-5x^2+x+2=(2x+1)(x-1)(x-2)$ ✓

1.2 1.2.1

Range $\{y \mid y\le 0\}$ or $y\le 0$ or $[0;\infty)$ ✓ (1)

1.2.2

$f(x)=x^2;\ x\le 0$

$x=y^2;\ y\le 0$ ✓ Interchange x and y

$y=-\sqrt{x}$ ✓ Making y subject; correct sign

$f(x)=-\sqrt{x}$ (2)

1.2.3

$g(x)=x-2$

$x=y-2$ ½ Interchange x and y

$y=x+2$ ½ Making y subject; correct sign

$g^{-1}(x)=x+2$ (1)

1.2.4

$f(x)=g^{-1}(x)\Rightarrow x^2=x+2$ ✓ Setting up equation

$x^2-x-2=0\Rightarrow (x+1)(x-2)=0$ ✓ Factorizing

$x=-1;\ 2$

Choose $x=-1$ ✓ Noting that *f* is defined for $x\le 0$

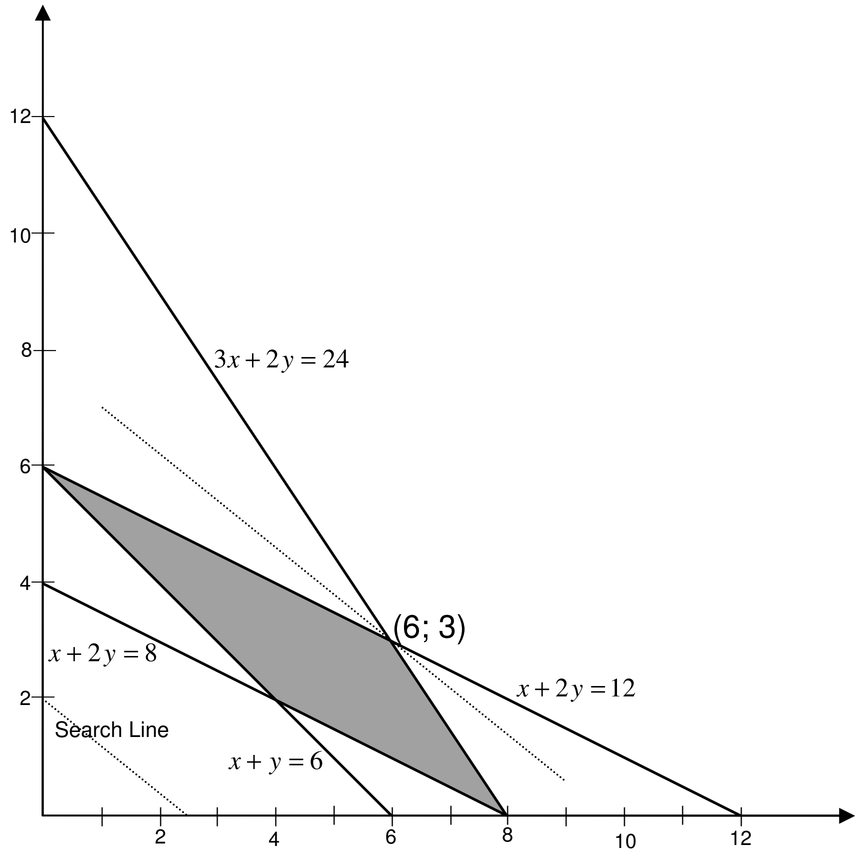
$y=f(-1)=(-1)^2=1$ or $y=g^{-1}(-1)=-1+2=1$ ✓

Point of intersection is (-1; 1)

Two points of intersection: 3 marks (4)

This document was downloaded from www.learnsupport.co.za
ACADEMIC SUPPORT TECHNOLOGIES (PTY) LTD

1.3 1.3.1



Mark scheme: Both axes ✓
First two straight lines correctly drawn (any two) ✓✓✓✓
Next two straight lines correctly drawn ✓✓
Feasible region shaded correctly✓ (8)

1.3.2 Objective unction: $y=-\frac{4}{5}x+\frac{P}{200}\Rightarrow P=160x+200y$ ✓ Making P the subject.
Profit per fax machine is R160 ✓
Profit per scanner is R200 ✓ (3)

1.3.3 Search line on graph or any other line with slope -4/5 ✓
Max Point: $x=6;\ \ y=3$ ✓
Production of 6 fax machines and 3 scanners ✓ (3)

1.3.4 Max Profit = 160(6) +200(3) =R1560 ✓✓ (2)

This document was downloaded from www.learnsupport.co.za
ACADEMIC SUPPORT TECHNOLOGIES (PTY) LTD

QUESTION 2

2.1 $f(x) = 2x^3$

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{2(x+h)^3 - 2x^3}{h}$ ✓✓ Formula; substitution

$= \lim_{h\to 0}\frac{2x^3 + 6x^2h + 6xh^2 + 2h^3 - 2x^3}{h}$ ✓ Expansion

$= \lim_{h\to 0}(6x^2 + 6xh + 2h^2)$ ✓✓ Simplify; dividing by h

$= 6x^2$ ✓ (6)

2.2.1 $y = (1-2x)^3$ ✓

$\frac{dy}{dx} = 3(1-2x)^2(-2)$ ✓

$= -6(1-2x)^2$ or $= -6 + 24x - 24x^2$ (2)

2.2.2 $f(x) = \frac{2x^3}{\sqrt{x}} + 10 = 2x^{5/2} + 10$ ✓ Simplification

$f'(x) = 2\left(\frac{5}{2}\right)x^{3/2} + 0$ or $f'(x) = 5x\sqrt{x}$ ✓ (2)

2.2.3 $y = x\ln x$ ✓ Simplification

$\frac{dy}{dx} = (1)\ln x + x\left(\frac{1}{x}\right) = \ln x + 1$ ✓ (2)

2.2.4 $y = \frac{e^{2x}+1}{2x^2-3}$ ✓ Simplification

$\frac{dy}{dx} = \frac{2e^{2x}.(2x^2-3) - 4x(e^{2x}+1)}{(2x^2-3)^2}$ ✓✓✓ Correct two terms in numerator and one in denominator (3)

2.3 2.3.1 $f(t) = -\sqrt{t}$ ✓

Velocity $= f'(t) = -\frac{1}{2\sqrt{t}}$

$f'(1) = -\frac{1}{2\sqrt{1}} = -\frac{1}{2}\ m/s$ ✓ (2)

2.3.2 $f'(t) = -\frac{1}{2\sqrt{t}}$

Acceleration

$= f''(t) = -\left(\frac{1}{2}\right)\left(-\frac{1}{t}\right)\left(\frac{1}{2\sqrt{t}}\right)$ ✓✓

$f''\left(\frac{1}{4}\right) = -\left(\frac{1}{2}\right)\left(-\frac{1}{\frac{1}{4}}\right)\left(\frac{1}{2\sqrt{\frac{1}{4}}}\right) = 2\ m/s^2$ ✓ Ignore units (3)

[20]

Copyright reserved **This document was downloaded from www.learnsupport.co.za** Please turn over

ACADEMIC SUPPORT
TECHNOLOGIES (PTY) LTD

QUESTION 3

3.1 3.1.1

$\int 4x(x-1)(x+1)dx$

$= \int (4x^3 - 4x)dx$

$= x^4 + 2x^2 + c$ ✓✓ Ignore "*c*" (2)

3.1.2

$\int \left(6e^{2x} + \frac{2}{x}\right)dx$

$= 3e^{2x} + 2\ln x + c$ ✓✓✓ One mark per term (3)

3.2 3.2.1

We find the x values of turning points of f.

$f(x) = x^3 - 3x^2 + c \Rightarrow f'(x) = 3x^2 - 6x$ ✓ Finding derivative

$3x^2 - 6x = 0 \Rightarrow x = 0;\ 2$ ✓ (3)

Minimum of f is at P on the x axis

$\therefore$ P has coordinates (2; 0) ✓

3.2.2

Substitute P(2; 0) into $y = x^3 - 3x^2 + c$

$0 = (2)^3 - 3(2)^2 + c$ ✓ Correct substitution

$c = 4$ ✓ (2)

3.2.3

Q(0; c) is the local maximum point

Coordinates of Q are (0; 4) (1)

3.2.4

Use P(2; 0) and Q(0; 4) to find m

$m = -\frac{4}{2} = -2$ ✓

$g(x) = -2x + 4$ ✓ (2)

3.2.5

$f'(x) = 3x^2 - 6x \Rightarrow f''(x) = 6x - 6$ ✓ For finding $f''(x)$

$6x - 6 = 0 \Rightarrow x = 1$

Point of inflection R has

coordinates $(1; f(1)) = (1;2)$ ✓ For coordinates (1;2) (2)

3.2.6

Area = $\int_0^1 \left(x^3 - 3x^2 + 4\right)dx$ ✓

$= \left[\frac{1}{4}x^4 - x^3 + 4x\right]_0^1$ ✓

$= (\frac{1}{4} - 1 + 4) - (0)$ ✓ (3)

= 3¼ square units

3.2.7

Shaded area = $\int_0^1 \left(x^3 - 3x^2 + 4\right)dx - \int_0^1 \left(-2x + 4\right)dx$ ✓

= 3¼ - $\left[-x^2 + 4x\right]_0^1$

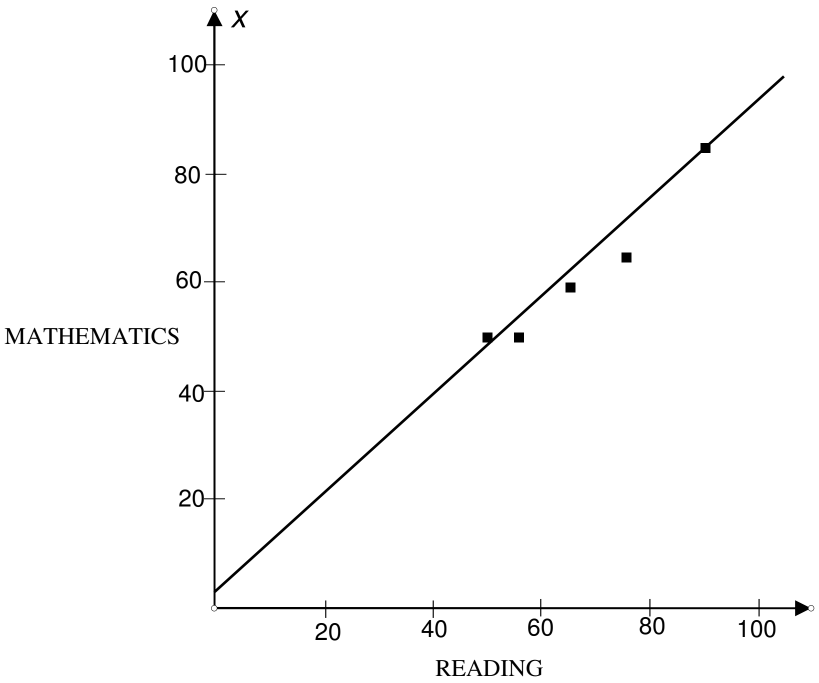
= 3¼ - 3 = ¼ square units ✓ (2)

[30]

This document was downloaded from www.learnsupport.co.za
ACADEMIC SUPPORT
TECHNOLOGIES (PTY) LTD

QUESTION 4

4.1 4.1.1 Mark scheme: ½ mark per point and ½ mark for both axes.



(3)

4.1.2 Line of best fit shown on graph (1)

4.1.3

A	B	C	D	E	F
$\sum x = 335$ ✓	$\overline{x} = 67$ ✓	$\sum y = 310$ ✓	$\overline{y} = 62$ ✓	$\sum (x-\overline{x})(y-\overline{y}) = 905$ ✓	$\sum (x-\overline{x})^2 = 1030$ ✓

(6)

4.1.4

$$b = \frac{\sum (x-\overline{x})(y-\overline{y})}{\sum (x-\overline{x})^2}$$ ✓

$= \frac{905}{1030} = 0,88$ [to 2 decimal places] ✓

$a = \overline{y} - b\overline{x} = 62 - 0,88(67) = 3,04$ [or 3,13] ✓

$y = 3,04 + 0,88x$ [or $y = 3,13 + 0,88x$] ✓ (4)

4.1.5 Substitute $x = 45$ in $y = 3,04 + 0,88x$ or $y = 3,13 + 0,88x$ ✓

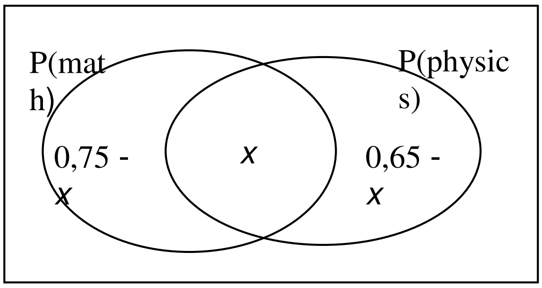
Dumi's Mathematics score will be $y = 3,04 + 0,88(45) = 43\%$ ✓ (2)

4.2 4.2.1 Let P(math) be the probability that Alfie passes mathematics and P(physics) be the probability that he passes physical science.

P(physics) = 1 – P(failing physics)
= 1 – 0,35 = 0,65 ✓ (1)

This document was downloaded from www.learnsupport.co.za
ACADEMIC SUPPORT TECHNOLOGIES (PTY) LTD

4.2.2 The probability of passing mathematics or physical science is 0,85.
Refer to the Venn diagram.
Let the probability of passing both mathematics and physical science be x. Then, from the diagram, we have:

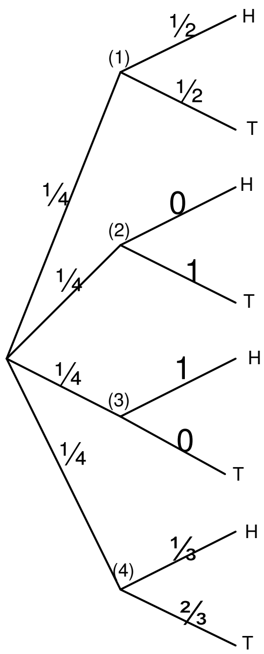


$0{,}75 - x + x + 0{,}65 - x = 0{,}85$ ✓✓
$x = 0{,}55$ ✓ (3)

4.2.3 P(failing math and physics) = 1 – 0,85 = 0,15 ✓ (1)

4.2.4 P(passing exactly one of math or physics) = P(math only) or P(physics only) ✓
= $(0{,}75 - x) + (0{,}65 - x)$ ✓
= 0,30 ✓ (3)

4.3



(a) P(coin 1 and T) = $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

(b) P(coin 2 and T) = $\frac{1}{4} \times 1 = \frac{1}{4}$

(c) P(coin 3 and T) = $\frac{1}{4} \times 0 = 0$

(d) P(coin 4 and T) = $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$

$\therefore$ P(any coin and T) = $\frac{1}{8} + \frac{1}{4} + 0 + \frac{1}{6} = \frac{13}{24}$

Mark scheme: 2 marks for correct tree
½ mark for each of (a), (b), (c) and (d) 2 marks for the answer.

(6)
[30]

TOTAL: 100

This document was downloaded from www.learnsupport.co.za
ACADEMIC SUPPORT
TECHNOLOGIES (PTY) LTD